Int. J. Heat Mass Transfer. Vol. 11, pp. 415–426. Pergamon Press 1968. Printed in Great Britain

TURBULENT HEAT TRANSFER IN ANNULI WITH SMALL CORES

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(*Received* 20 *April* 1967 *and in revised form* 16 *August* 1967)

Abstract—The j factors for heat and momentum and the maximum points of the velocity profiles have been experimentally determined for air flow at Reynolds number from 17000 to 100000 in a tube and three concentric annuli having radius ratios of about 10, 16 and 31. The heat-transfer data are correlated successfully by means of geometrical transformations which yield coincident patterns of the eddy diffusivities for momentum.

NOMENCLATURE

C, constant coefficient in equation (21) [dimensionless];
D, inside diameter of tube [ft];
D_1, inner diameter of annulus [ft];
D_2, outer diameter of annulus [ft];
D_e, equivalent diameter of annulus, $= (D_2 - D_1)$ [ft];
F, function defined by equation (1) [dimensionless];
F_1, function defined by equation (2) [dimensionless];
F_2, function defined by equation (3) [dimensionless];
G, mass velocity of fluid [$lb_m/s\,ft^2$];
j_F, j factor for friction at tube wall [dimensionless];
j_{F_1}, j factor for friction at inner wall of annulus [dimensionless];
j_{F_2}, j factor for friction at outer wall of annulus [dimensionless];
j_H, j factor for heat transfer at tube wall [dimensionless];
j_{H_1}, j factor for heat transfer at inner wall of annulus [dimensionless];
j_{H_2}, j factor for heat transfer at outer wall of annulus [dimensionless];
m, constant exponent in equation (21) [dimensionless];
n, constant exponent in equation (21) [dimensionless];
N_{Pr}, Prandtl number of fluid [dimensionless];
N_{Re}, Reynolds number [dimensionless];
N_{Re_a}, Reynolds number of annulus, defined by equation (6) [dimensionless];
N_{Re_t}, Reynolds number of tube equivalent to outer portion of annulus, defined by equation (7) [dimensionless];
N_{Re_p}, Reynolds number of tube equivalent to inner portion of annulus, defined by conditions of equation (11) [dimensionless];
$N^{**}_{Re_t}$, modified Reynolds number defined by equation (15) [dimensionless];
$N^{**}_{Re_p}$, modified Reynolds number defined by equation (16) [dimensionless];
N_{θ_1}, function defined by equation (17) [dimensionless];
N_{θ_2}, function defined by equation (18) [dimensionless];
N_τ, function $= (\Phi_{m2}R_{H_2}/\Phi_{m1}R_{H_1})_L \times (\Phi_{m1}R_{H_1}/\Phi_{m2}R_{H_2})$;
r_1, inner radius of annulus [ft];
r_2, outer radius of annulus [ft];

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r_m, radius of maximum velocity in annulus regardless of type of flow [ft];
r_{mL}, radius of maximum velocity in annulus when flow is entirely laminar [ft];
r_θ, radius of minimum temperature in annulus [ft];
R_0, radius of equivalent tube, defined by equation (7) [ft];
R_{H_1}, hydraulic radius for inner portion of annulus $= (r_m^2 - r_1^2)/2\,r_1$ [ft];
R_{H_2}, hydraulic radius for outer portion of annulus $= (r_2^2 - r_m^2)/2\,r_2$ [ft];
t_1, temperature at inner wall of annulus [°F];
t_2, temperature at outer wall of annulus [°F];
t_m, minimum local temperature in annulus [°F];
u_m, maximum local velocity [ft/s];
V, bulk average linear velocity [ft/s].

Greek symbols
μ, fluid viscosity [lb$_m$/s ft];
ρ, fluid density [lb$_m$/ft^3];
Φ_{m1}, function $= \dfrac{1}{2R_{H_1}^2}\left(r_1^2 - r_m^2 - 2r_m^2 \ln \dfrac{r_1}{r_m}\right)$ [dimensionless];
Φ_{m2}, function $= \dfrac{1}{2R_{H_2}^2}\left(r_2^2 - r_m^2 - 2r_m^2 \ln \dfrac{r_2}{r_m}\right)$ [dimensionless];
ψ, function of $(t_1 - t_m)/(t_2 - t_m)$ which accounts for differences in inner and outer wall temperatures of an annulus [dimensionless]. For heating at both walls with the ratio of heat fluxes equal to the ratio of skin frictions, $\psi = (t_1 - t_m)/(t_2 - t_m)$. For heating at the inner wall only, $\psi = 1$.

Subscripts
1, inner wall or inner portion of annulus;
2, outer wall or outer portion of annulus;
a, refers to annulus;
f, refers to "film" conditions;
p, t, refer to equivalent tubes;
L, quantities evaluated with r_m replaced by r_{mL}.

THE j FACTORS for heat and momentum associated with turbulent flow in smooth tubes stand in the ratio

$$j_H/j_F = F(N_{Re_f}, N_{Pr_f}). \tag{1}$$

Similarly, in a smooth concentric annulus of inner radius r_1 and outer radius r_2,

$$j_{H_1}/j_{F_1} = F_1(N_{Re_{f1}}, N_{Pr_{f1}}, r_2/r_1) \tag{2}$$

$$j_{H_2}/j_{F_2} = F_2(N_{Re_{f2}}, N_{Pr_{f2}}, r_2/r_1). \tag{3}$$

Deissler [1] and others have shown that experimental data for smooth tubes are represented only roughly when the function F is taken to be unity as suggested by Colburn [2]. The simplicity of the resulting "analogy" is attractive, however, for practical reasons. Knudsen [3] has examined data on annuli from several sources and has concluded that most can be represented to ± 10 per cent by taking the functions F_1 and F_2 to be unity. He has also observed, however, that the agreement becomes significantly less satisfactory as the radius ratio r_2/r_1 is increased. It is to the point, therefore, to gain a more satisfactory basis for handling large radius ratios while preserving the convenient form of the preceding equations.

When the radial distributions of the eddy diffusivities for heat and momentum are completely predetermined, the pertinent j-factor ratio can be obtained directly. The same is true when only the eddy diffusivity for momentum is known but a reasonable assumption can be made about its relationship to the eddy diffusivity for heat. Tubes have been quite adequately handled thereby [1, 4] with numerous supporting data, but annuli have not. For the present, it remains appropriate to deal with annuli in terms of "equivalent" tubes.

In an annular duct there are two boundaries to consider and therefore two equivalent tubes to specify. The portions of the stream inside and outside the radius of maximum velocity have to be treated individually. For smooth walls, reference to an equivalent tube implies two specifications, namely the size of the tube and its operating Reynolds number. Even when the same size of tube is chosen to represent both parts of an annulus, the Reynolds number equivalent to the inner portion may not be the same as that equivalent to the outer one.

It is reasonable to presume that the proper equivalent tube might be the one having the same velocity pattern as the actual annulus on suitably transformed coordinates. The profiles of diffusivities for momentum are thus rendered coincident, leaving the way open for study of the heat transfer under comparable hydrodynamic conditions. Sufficient velocity data for non-isothermal flow are lacking, but the criteria for equivalence in isothermal cases have been developed previously [5].

FRICTION

In fully laminar, isothermal flow, the radius of maximum velocity r_{mL} is at the position

$$r_{mL} = [(r_2^2 - r_1^2)/2 \ln (r_2/r_1)]^{\frac{1}{2}}. \tag{4}$$

In the turbulent range, the isothermal maximum point r_m depends on the Reynolds number as well as on the radius ratio. At high Reynolds numbers, however, the dependence on the latter prevails [5, 6] and

$$r_m = r_1 + \left(\frac{r_2 - r_1}{2}\right)\left(\frac{r_1}{r_2}\right)^{0\cdot 20}. \tag{5}$$

When the flow is laminar, the tubes representing the inner and outer portions of the annulus have the same size and operate at the same Reynolds number. On the other hand, when the flow is highly turbulent and the maximum point lies closer to the core, the two equivalent tubes have equal size but the one representing the inner portion of the annulus operates at a higher Reynolds number than the one representing the outer portion.

It is convenient to define the Reynolds number N_{Re_a} for the annulus as

$$N_{Re_a} = (2\Phi_{m2} R_{H2}) V_a \rho/\mu. \tag{6}$$

The tube representing the outer portion of the annulus has the radius R_0 and the Reynolds number N_{Re_t} given by the following equations:

$$R_0 = (\Phi_{m2} R_{H2}) \tag{7}$$

$$N_{Re_t} = 2R_0 V_t \rho/\mu. \tag{8}$$

The Reynolds numbers are connected through the simple criterion

$$N_{Re_t}(u_m/V)_t = N_{Re_a}(u_m/V)_a \tag{9}$$

and the j_F factors are related by the stipulation that

$$j_{F_t}(V/u_m)_t^2 = j_{F_2}(V/u_m)_a^2. \tag{10}$$

The tube representing the inner portion of the annulus also has the radius R_0 specified in equation (7), but it operates at a Reynolds number N_{Re_p} such that

$$j_{F_p}(V/u_m)_p^2 = N_\tau j_{F_2}(V/u_m)_a^2 \tag{11}$$

where

$$N_\tau = \left(\frac{\Phi_{m2} R_{H2}}{\Phi_{m1} R_{H1}}\right)_L \left(\frac{\Phi_{m1} R_{H1}}{\Phi_{m2} R_{H2}}\right).$$

The subscript L means that the quantities in the first bracket are calculated with r_m replaced by r_{mL} from equation (4). The other symbols and subscripts are shown in the Nomenclature. To complete the picture, the j_F factors at the two surfaces of the annulus are related through the elementary force balance:

$$\frac{j_{F_1}}{j_{F_2}} = \frac{r_2}{r_1}\left[\frac{(r_m^2 - r_1^2)}{(r_2^2 - r_m^2)}\right]. \tag{12}$$

When the flow is nonisothermal, the friction is affected by variations in the fluid properties and by displacement of the maximum point from its isothermal position. The actual value of r_m for the case at hand should therefore be used whenever possible and, in keeping with certain practice, the Reynolds number may be evaluated at the "film" temperature associated with the particular boundary being considered.

HEAT TRANSFER

Since heat can be transferred across either or both boundaries of an annulus, any number of temperature patterns can co-exist with a particular velocity profile. Depending on the magnitudes and directions of the surface heat fluxes, there may or may not be a minimum or maximum of local temperature within the stream. The present considerations are limited to certain cases in which there is an input of heat through one or both of the boundaries and, therefore, there is a minimum temperature at some point within the annulus.

Case 1. Input at both walls; special flux ratio

A straightforward situation occurs when heat is transferred into a gas from both walls at just the rates needed to produce the minimum temperature exactly at the radius of maximum velocity. In that case, equations (10–12) afford an obvious means of dealing with the heat transfer. They assert that the hydrodynamic equivalence of tubes and annuli depends on the parameter $j_F(V/u_m)^2$. By virtue of equation (1), the j-factor ratio in smooth tubes depends on the same parameter once a particular fluid is specified. That implies that the local eddy diffusivities for heat and momentum follow the same parametric pattern in tubes and annuli. Thus when the point of minimum temperature and maximum velocity occur at the same radius, equations (10) and (11) express the conditions under which equal j_H values can be expected when comparing the two types of conduits.

It follows directly that at the outer wall,

$$j_{H_2} = F j_{F_2}(V/u_m)_a^2/(V/u_m)_t^2 \qquad (13)$$

and at the inner wall,

$$J_{H_1} = F(j_{F_1}/j_{F_2})^* N_\tau j_{F_2}(V/u_m)_a^2/(V/u_m)_p^2 \qquad (14)$$

where

$$(j_{F_1}/j_{F_2})^* = (j_{F_1}/j_{F_2})(t_1 - t_m)/(t_2 - t_m).$$

The ratio of temperature differences $(t_1 - t_m)/(t_2 - t_m)$ is simply a scale factor to account for the difference in temperature levels between the inner and outer walls. The velocity profiles related to j_{F_1} and j_{F_2} have the same scale, namely $(u_m - 0)$, while the temperature profiles are scaled on $(t_1 - t_m)$ and $(t_2 - t_m)$ respectively. When the fluid has a Prandtl number of unity and when the minimum temperature and maximum velocity occur at the same point in the stream, the proper scale factor is the ratio of maximum temperature differences as shown, but the general case is not as simple.

Case 2. Input at both walls; general

When the heating occurs at any other relative rates, the minimum temperature is situated at some radius r_θ which is different than the radius of maximum velocity r_m. The temperature profiles inside and outside the minimum point therefore occupy the radial distances $(r_\theta - r_1)$ and $(r_2 - r_\theta)$ respectively, while the velocity profiles extend over the corresponding distances $(r_m - r_1)$ and $(r_2 - r_m)$. The most obvious effect is to change the effective Reynolds numbers of the equivalent tubes from the viewpoint of the temperature distribution. Since the parameter $j_F(V/u_m)^2$ is only a weak function of the Reynolds number, it is satisfactory in the absence of better information to take the new Reynolds numbers as follows:

$$N_{Re_t}^{**} = N_{Re_t}(r_2 - r_\theta)/(r_2 - r_m) \qquad (15)$$

$$N_{Re_p}^{**} = N_{Re_p}(r_\theta - r_1)/(r_m - r_1). \qquad (16)$$

In line with the same reasoning as before, the new j_H factors should behave as dictated by the altered values of $j_F(V/u_m)^2$ corresponding to the new Reynolds numbers. Therefore it is convenient to introduce dimensionless ratios of the skin friction parameters:

$$N_{\theta 1} = [j_{F_p}(V/u_m)_p^2]^{**}/[j_{F_p}(V/u_m)_p^2] \qquad (17)$$

$$N_{\theta 2} = [j_{F_t}(V/u_m)_t^2]^{**}/[j_{F_t}(V/u_m)_t^2]. \qquad (18)$$

Then by virtue of equations (13) and (14),

$$j_{H_2} = FN_{\theta_2} j_{F_2} (V/u_m)_a^2/(V/u_m)_t^2 \quad (19)$$

$$j_{H_1} = FN_{\theta_1} (j_{F_1}/j_{F_2})^* N_r j_{F_2} (V/u_m)_a^2/(V/u_m)_p^2 \quad (20)$$

where

$$(j_{F_1}/j_{F_2})^* = (j_{F_1}/j_{F_2})\, \psi[(t_1 - t_m)/(t_2 - t_m)].$$

The function ψ cannot be evaluated on the basis of present information when r_θ and r_m are different. Cases 1 and 3 are therefore the only ones which can be treated quantitatively. Nonetheless, equations (19) and (20) are the sought-for relationships to be tested by experiment. They are of course limited to cases for which neither heat-transfer coefficient is negative when defined in the usual way.

Case 3. *Input at one wall only*

If one of the walls is adiabatic, the preceding equation for the heated wall still applies. When the outer wall has no heat flow associated with it, $(r_\theta - r_1)$ becomes $(r_2 - r_1)$ and equation (20) describes the transfer at the core. When the inner wall is adiabatic, $(r_2 - r_\theta)$ becomes $(r_2 - r_1)$ and equation (19) describes the transfer at the outer wall.

In this case there can be no effect of the scale factor ψ since only one portion of the temperature profile exists. Therefore ψ must be equal to unity in the limit and $(j_{F_1}/j_{F_2})^* = (j_{F_1}/j_{F_2})$ when only the inner wall of the annulus is heated.

SCOPE OF EXPERIMENTS

Uncertainty about the effect of the Prandtl number tends to mask and distort the influence of geometry, so air was used as the test fluid to minimize the effects of temperature gradients on properties other than density. The radius of maximum velocity was measured in order to permit partitioning of the individual skin frictions in both isothermal and heating runs. To provide data in a range where they are sparse, three annuli having radius ratios of roughly 10, 16 and 31 were formed by inserting appropriate cores into the same outer tube. The tube was also operated separately without a core. In each case, all of the pertinent j factors were determined simultaneously in order to furnish a consistent set of data for comparison. Reynolds numbers were in the range of 17000 to 100000, expressed on the basis of the usual over-all equivalent diameter. The experimental runs included operation with no heating, with both walls heated and with either the inner or outer wall heated while the other wall was kept adiabatic.

EXPERIMENTAL EQUIPMENT AND PROCEDURE

The flow system provided a once-through path for room air to move in series through 50 ft of inlet pipe, a positive-displacement blower, a surge tank, a standard 2-in, flange-tap orifice meter, a preheater consisting of 6 ft of pipe wrapped with nichrome ribbon, the test section, and thence to the outside atmosphere.

The test section was basically a horizontal, cold-drawn, seamless, carbon steel tube 41-ft long with an inside diameter of 3·015 in. The tube was heated by tape-covered nichrome ribbon wrapped around it in six $5\frac{1}{2}$-ft sections with separately controlled electrical inputs. It was thoroughly insulated from the surrounding air by two layers of aluminum foil, a vacuum jacket and 3 in of asbestos lagging.

Three sizes of cores were used to form the concentric annuli. All were 304 stainless steel tubes 39-ft long with a bright 63 μ-in finish. Their outside diameters were 0·3156, 0·1890 and 0·0959 in yielding radius ratios of 9·55, 15·95 and 31·44 respectively when installed in the outer tube. The cores were heated by passing alternating current through their walls. They were centered by a combination of tension and mechanical support, the latter in the form of eight fin-type, insulated assemblies placed $4\frac{1}{2}$ ft apart. The maximum sag in any case was about 12 mm.

The air, regulated to 0·1 degF by the preheater, entered the annular section from a baffled inlet jacket through forty-five screen-covered $\frac{1}{2}$-in holes in the outer tube. At the other end of the test section, a similar installation with eighty-seven holes but no baffles or screens served as the outlet.

At a point 30 ft 4 in downstream from the end of the inlet jacket, a double impact tube was inserted through the top of the outer tube for the purpose of determining the radius of maximum velocity. The calming lengths were roughly one hundred and thirty equivalent diameters upstream from the impact tube and thirty-seven downstream. The probe consisted of concentric stainless steel tubes with outside diameters of 0·049 and 0·095 in terminating in separate openings of 0·033 in dia. spaced 0·107 in apart along the radius of the annulus. The other end of the probe permitted the impact pressures felt by the two tubes to be transmitted separately to the opposite sides of a differential micromanometer. The probe assembly, about 10-in long in all, was attached to a traversing mechanism capable of positioning it to 0·001 in on the radius of the annulus. The micromanometer, a modified tilting type, was calibrated to 0·0001 in of water.

The outer tube wall temperature was measured by thermocouples at twelve positions 3 ft apart. The core wall temperature was measured at two points on the periphery in the plane of the impact tube. The thermocouple leads were brought out through opposite ends of the core and the lead wires were twisted together to eliminate induction of an alternating current in the thermocouple circuits. Static pressure taps were installed in the outer tube at the impact probe and $2\frac{1}{2}$ ft on either side of it. The separate impact pressures were measured on an inclined manometer and the orifice pressure drop, upstream orifice pressure and the static pressure at the impact tube were measured with U-tube manometers. Static pressure drops due to friction were measured by means of the micromanometer.

Core currents and voltages were measured to 0·4 A and 0·4 V or better. Outer wall voltage was measured to 0·75 V in all six heating sections and the corresponding currents were measured to 0·2 A. Thermocouple voltages were measured with a Type K Leeds and Northrup potentiometer.

Calculations were programmed on a digital computer. Allowance was made for end effects, external heat losses and the influence of density changes on the measurements of skin friction and the radius of maximum velocity. The double impact tube was calibrated to allow for differences in the separate impact coefficients and then was used to traverse the vicinity of the maximum point. The temperature differences between the walls and the bulk of the fluid ranged from 10 degF to 100 degF, in round numbers, with most between 20 degF and 50 degF.

RESULTS AND DISCUSSION

Figure 1 shows how the radius of maximum velocity is affected by the Reynolds number and by the transfer of heat. The ordinate is left dimensional to keep the three sets of curves separated from one another for easier viewing. When the flow of the gas is isothermal, the maximum point is affected very little by the Reynolds number and its position at the higher end of the investigated range is in excellent agreement with equation (5). In nonisothermal cases where the gas is heated at one wall while the other wall remains adiabatic, the maximum point is displaced away from the heated wall, an effect most pronounced at low Reynolds numbers.

To put the results in a form directly comparable to older correlations, use can be made of the Reynolds number defined on the over-all equivalent diameter $D_e = (D_2 - D_1)$ and the various j factors can be expressed as

$$j = C(D_2/D_1)^m (D_e G/\mu_f)^n. \qquad (21)$$

The correlations for isothermal flow and for heating at one wall only are summarized in Table 1. There were not enough data on simultaneous heating at both walls to permit similar treatment of that case.

For the outer tube operated without a core,

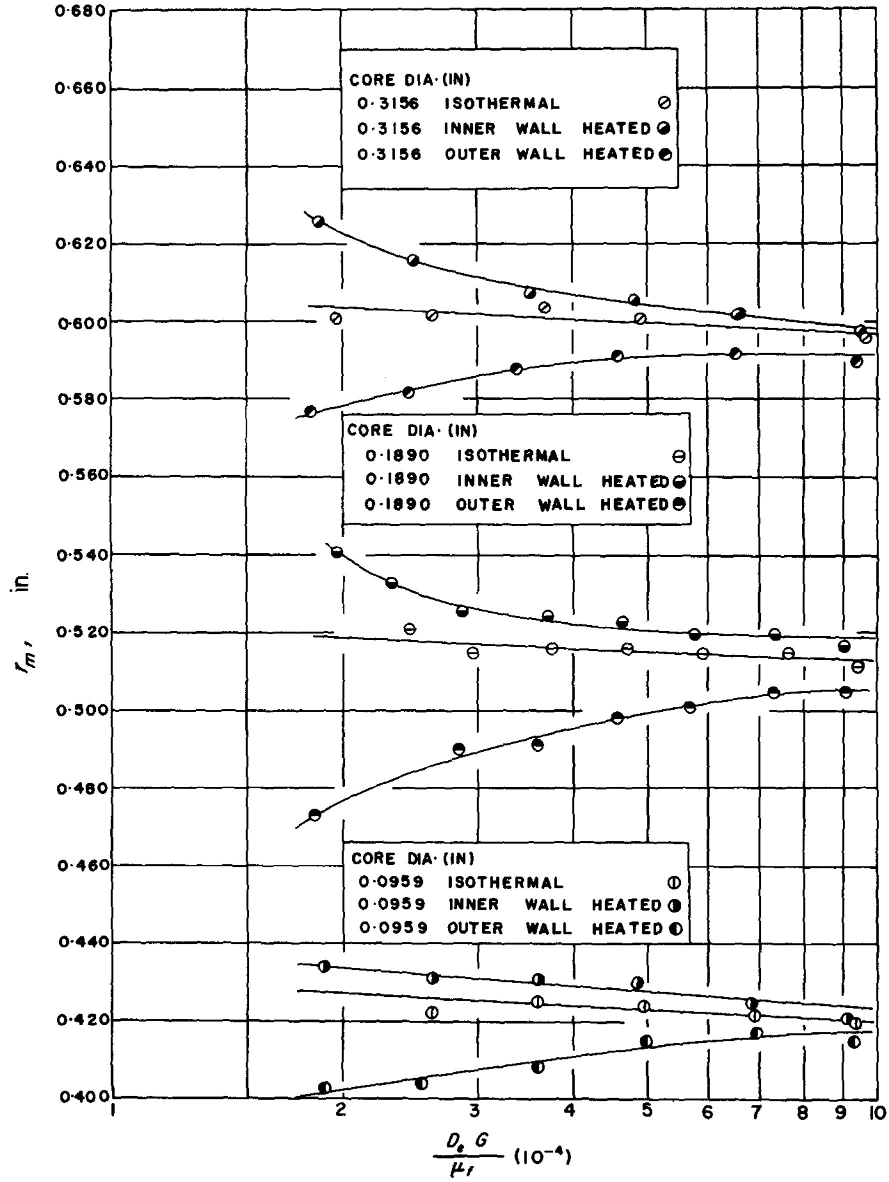

FIG. 1. Effect of heat transfer on radius of maximum velocity in experimental annuli.

the ratio of j factors is influenced very little by the Reynolds number. Thus, for all practical purposes equation (1) can be written with

$$F = 0{\cdot}81 \pm 0{\cdot}01$$

based on eleven points determined exclusively for the heated tube. In the present case, then, an error of about 24 per cent in j_H is incurred by taking F to be unity. Likewise, the same error results in j_{H_2} for outer wall heating in the three annuli. For heating of the inner wall alone, the corresponding errors in j_{H_1} amount

Table 1. Summary of experimental results: $j = C(D_2/D_1)^m (DeG/\mu_f)^n$

Type of duct	Heating condition	j	C	m	n	No. of exponential points	Average precision ($\pm$ %)
Tube and annuli	isothermal and all types of heating	j_F and j_{F2}	0·034	0	$-0·23$	78	1·1
Annuli	isothermal	j_{F1}	0·027	0·40	$-0·25$	59	0·8
Tube and annuli	outer wall heating only	j_H and j_{H2}	0·0193	0	$-0·20$	30	1·4
Annuli	inner wall heating only	j_{H1}	0·023	0·25	$-0·25$	17	1·2

to anywhere from 60 per cent to 100 per cent depending on the radius ratio when the calculation is based on the isothermal value of j_{F_1} as usual. Thus, while deviations at the outer wall may be largely attributable to the behavior of the outer tube itself, the same cannot be claimed at the inner wall, especially when the radius ratio is large.

The j_F factors for the experimental tube without a core are about 5 per cent greater than predicted by the Blasius equation. The tendency for air flow to yield slightly higher values than liquids at Reynolds numbers above 20000 has been observed previously. The data of Stanton and Pannell [7] show j_F values for air flow to be about 4 per cent greater than the Blasius prediction in tubes and the data of Brighton and Jones [6] suggest that the same is true in annuli of various radius ratios.

Regarding the j_H factors for air flow in tubes, Drexel and McAdams [8] correlated what they judged to be the most reliable data in the literature to ± 25 per cent by an equation which for a Prandtl number of 0·70 takes the form

$$j_H = 0{\cdot}0204(DG/\mu_f)^{-0{\cdot}20}. \qquad (22)$$

The present results for the outer tube alone are therefore only about 5 per cent lower than predicted by Drexel and McAdams, well within the stated precision of equation (22).

Three runs for each core size were made while heating at both walls with the heat fluxes in roughly the same ratio as the skin frictions. Under such conditions, the minimum temperature and maximum velocity should occur at approximately the same radius, thus satisfying a condition under which simple analogies are most likely to be valid. The radius of maximum velocity lay between the isothermal value and that for heating the outer wall alone, but closer to the isothermal position. As expected, the j_{H_2} values were only about 18 per cent less than predicted by taking F_2 of equation (2) to be unity, but the j_{H_1} factors were 25 per cent to 67 per cent in error with some indication of a dependence on the radius ratio. The results were therefore consistent with those obtained while heating one wall at a time.

CORRELATION OF RESULTS

By virtue of equations (9) and (10), the graph of $j_F(V/u_m)^2$ against $N_{Re_f}(u_m/V)$ uniquely relates the j_F factors in tubes to those at the outer walls of annuli. Figure 2 shows the correlation for isothermal flow in the outer tube and all three annuli. Figure 3 is the same kind of graph for the runs involving heat transfer at the outer wall. It is evident that the same solid line represents both sets of data very closely.

With the j factors for friction established, equations (19) and (20) can be graphed on the same basis, as shown in Figs. 4 and 5. The curve representing the frictional data is also an excellent representation of the heat-transfer data and the prediction of j_{H_1} is especially improved. The greater scatter of the j_{H_1} points is largely due to

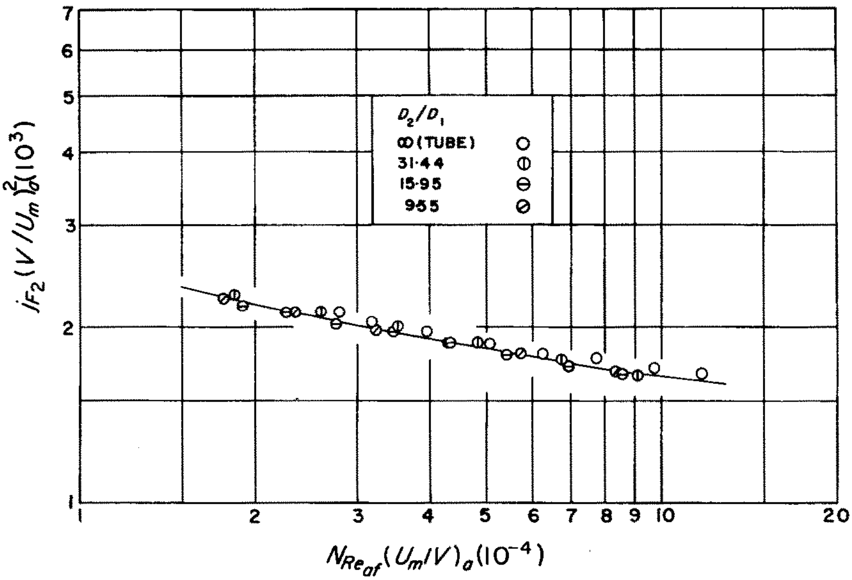

FIG. 2. Correlation of j_F for tube and for outer walls of annuli; isothermal flow.

the fact that the calculations amplify the effects of experimental uncertainties, particularly with regard to the radius of maximum velocity. All of the points shown in Figs. 2–5 are computed from unsmoothed data.

The radius of minimum temperature r_θ was not measured and therefore had to be approximated when heat was transferred simultaneously at both walls. For that purpose, the mass flow rates on each side of the point of maximum velocity were combined with temperature and heat flux data to yield rough energy balances

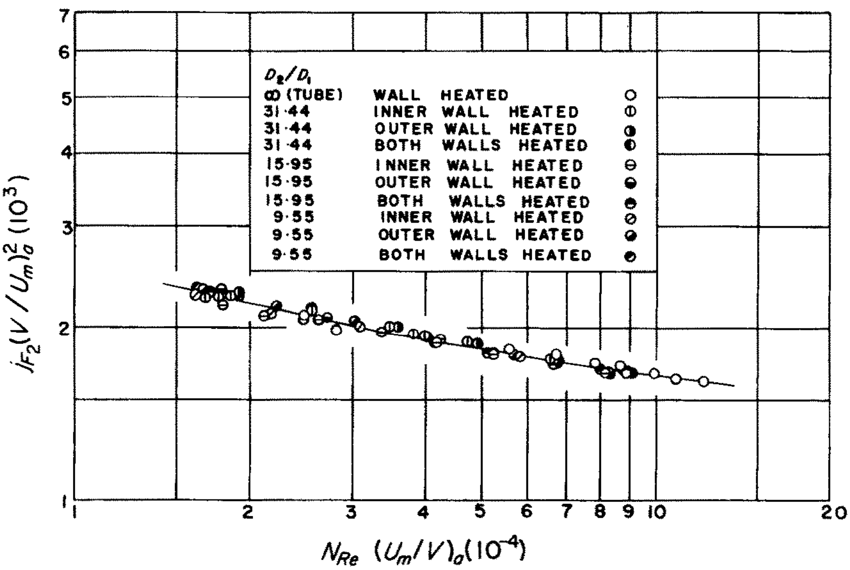

FIG. 3. Correlation of j_F for tube and for outer walls of annuli; heating at one wall and at both walls.

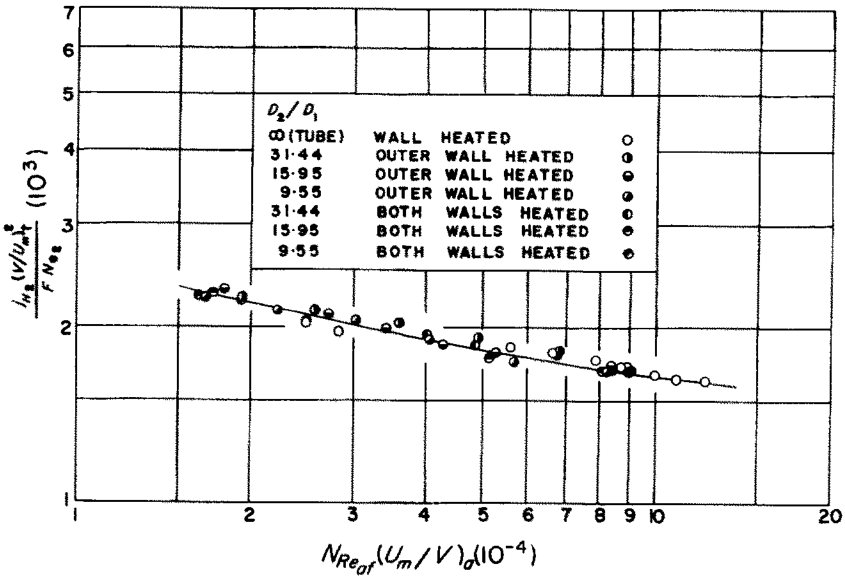

FIG. 4. Correlation of j_H for tube and for outer walls of annuli; heating at outer wall only and at both walls. (Solid line shows prediction based on j_F.)

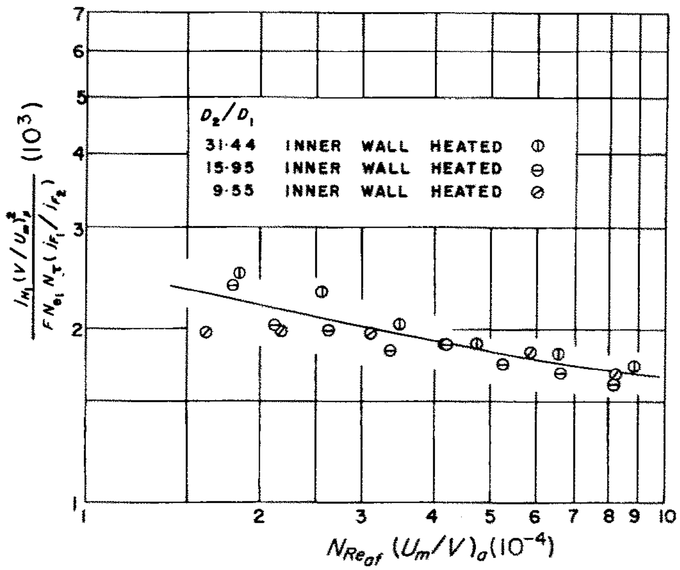

FIG. 5. Correlation of j_H for inner walls of annuli; heating at inner wall only. (Solid line shows prediction based on j_F.)

from which an approximate value of r_θ was calculated. Since the temperature of the inner and outer walls were different, the scale factor $(t_1 - t_m)/(t_2 - t_m)$ was introduced even though r_θ and r_m did not quite coincide. The minimum temperature t_m was approximated by taking the ratio of average to maximum temperature differences to be equal to $(V/u_m)_a$. Even with these assumptions and approximations, Table 2 shows the correlation to be very satisfactory

Table 2. Summary of experimental j_{H1} values for heating at both walls

$\frac{D_2}{D_1}$	$t_1 - t_m$	$t_2 - t_m$	$\frac{t_1 - t_m}{t_2 - t_m}$	$j_{F2}(V/u_m)_a^2$	$\frac{j_{H1}(V/u_m)_a^2}{FN_{\tau}N_{\theta 1}(j_{F1}/j_{F2})^*}$	% deviation
31·44	25·9	24·3	1·07	0·00228	0·00273	+20·0
31·44	30·5	24·6	1·24	0·00187	0·00186	−0·5
31·44	36·8	28·0	1·32	0·00166	0·00162	−2·4
15·95	47·9	35·8	1·34	0·00230	0·00235	+2·2
15·95	56·9	43·4	1·31	0·00181	0·00173	−4·4
15·95	50·7	33·2	1·53	0·00165	0·00161	−2·4
9·55	24·4	19·5	1·25	0·00234	0·00263	+12·3
9·55	25·4	21·6	1·18	0·00192	0·00192	0·0
9·55	30·3	27·6	1·10	0·00168	0·00173	+3·0

except at the lowest Reynolds numbers where r_θ is most in question.

It can be concluded that whatever the j_H/j_F ratio for the outer tube alone, the geometrical effects associated with extending it to an annulus can be handled by the relationships developed herein. On the other hand, the influence of variable fluid properties remains an open question and any extension to systems other than ordinary gases should be made with caution.

A more complete development of the method for obtaining j factors for friction is given in [5]. Values of V/U_m in annuli for use in conjunction with Figs. 2 and 3 are presented in [9]. Values of V/U_m in smooth tubes to be used in conjunction with Figs. 4 and 5 are also included in [9].

ACKNOWLEDGEMENT

The data appear in the thesis, "Heat and Momentum Transfer for Turbulent Flow in Annuli with Small Cores", submitted in 1966 by Reid B. Crookston in partial fulfilment of the requirements for the degree of Doctor of Philosophy at Carnegie Institute of Technology. The thesis is available on interlibrary loan from the Hunt Library, Carnegie Institute of Technology, Pittsburgh, Pa. 15213. Fellowship support from the Cities Service Company is gratefully acknowledged.

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Résumé—Les facteurs j pour la chaleur et la quantité de mouvement et les maxima des profils de vitesse ont été déterminés expériementalement pour un écoulement d'air à des nombres de Reynolds de 17 000 à 100 000 dans un tube et trois tuyaux annulaires concentriques ayant des rapports de rayons d'environ 10, 16 et 31. Les résultats de transport de chaleur sont corrélés avec succès au moyen de transformations géométriques qui font coïncider les distributions de diffusivité turbulente pour la quantité de mouvement.

Zusammenfassung—Die j-Faktoren für den Wärme- und Impulsaustausch und die Maxima der Geschwindigkeitsprofile wurden an Luftströmungen im Reynolds-Zahlenbereich von 17 000 bis 100 000 in einem Rohr und drei Ringräumen von Durchmesserverhältnissen von etwa 10, 16 und 31, experimentell ermittelt. Die Wärmeübergangsdaten wurden mit Erfolg korreliert auf Grund von geometrischen Transformationen, die übereinstimmende Muster für die turbulenten Austauschkoeffizienten ergaben.

Аннотация—Экспериментально определялись *j*–коэффициенты для тепла и количества движения и максимальные точки профилей скорости для потока воздуха $Re = 17000$–100 000 в трубе и трех концентрических каналах, имеющих отношение радиусов 10, 16 и 31. Данные по теплообмену хорошо описываются с помощью геометрических преобразований, которые дают картины совпадения коэффициентов турбулентной диффузии для переноса количества движения.